Question 1

(a) With the diaphragm being at 0.0 cm, we compare the first column with the differences between the columns, eg, 6.8 with (13.8 – 6.8) = 7.0 cm; 5.3 with 5.1 and 5.1 etc and find them equal within the ± 0.2 cm uncertainty of the difference. So we expect the diaphragm to behave very nearly as a displacement node

(b) We would expect the rigid clamp to be a displacement node.

(c) Including the diaphragm and clamp positions as well as the observed nodes, we get:

Frequency / Hz	Internodal distances / cm	Average / cm
290	6.8 7.0 7.2	7.0 ± 0.2
390	5.3 5.2 5.1 5.4	5.3 ± 0.2
490	4.3 4.1 4.2 4.1 4.3	4.2 ± 0.2
580	3.4 3.6 3.5 3.3 3.5 3.7	3.5 ± 0.2

(d) internodal distance = ½ λ

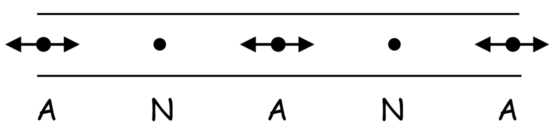
(e)

Frequency f / Hz ± 5 Hz	Average internodal distances / cm ± 0.2 cm	Wavelength λ / cm ± 0.4 cm	Velocity v / ms^{-1} ± 2 ms^{-1}
290	7.0	14.0	40.6
390	5.3	10.6	41.3
490	4.2	8.4	41.2
580	3.5	7.0	40.6

The percentage uncertainties in the velocity calculated from the uncertainties in frequency and wavelength range from 4.6 to 6.6%; say 5½% of 40 giving ± 2 ms^{-1}. The spread in the velocity values is less than this; so the velocity is best stated as v = 41 ± 2 ms^{-1}.

(f) Each part of the spring between adjacent nodes does a longitudinal simple harmonic motion with the same frequency and phase but an amplitude that depends on its position. (The SHMs on either side of a node are out of phase by 180°.)

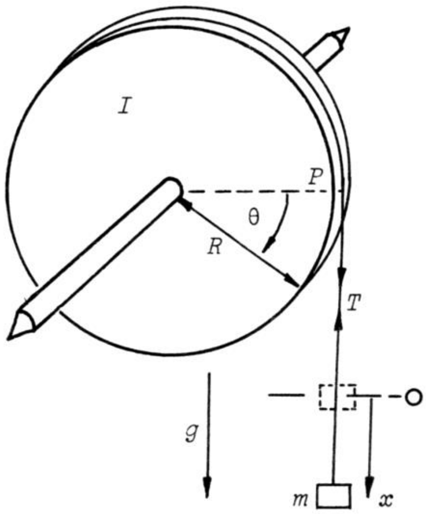
(g) Open-ended air column
A antinode, N node



Question 2

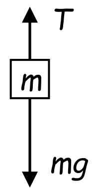
(a) Because the wheel's weight and the axle support force act through the axis and therefore have zero lever arm about the axis, they exert no torque on the wheel. Only the off-centre force exerted on the wheel rim by the string exerts a torque.
The string force is the string tension T; the lever arm is the wheel radius R;
thus torque = string tension × lever arm

$$\tau = TR$$



(b) Newton's second law for rotational motion
$\Rightarrow$ total torque acting = rotational inertia I × consequent angular acceleration α
$\Rightarrow \tau = TR = I\,\alpha$
$\Rightarrow$ ang acceln of wheel is $\alpha = \dfrac{TR}{I}$

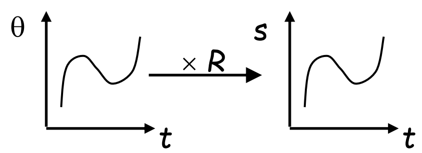
(c) Forces acting on the mass are its downward weight mg and the upward support force of the string tension T. Since the mass accelerates downward, its weight exceeds the string tension
$\Rightarrow$ net downward force on the mass is $mg - T$.



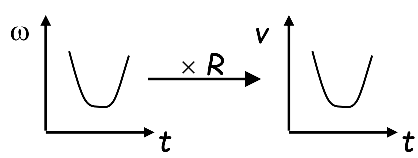
(d) Newton's second law for linear motion
$\Rightarrow$ total force acting F = mass m × consequent linear acceleration a
$\Rightarrow$ F = mg – T = m a
$\Rightarrow$ linear acceln of wheel is $a = \dfrac{mg - T}{m}$

(e) From definition of radian measure, angle θ at centre of circle subtends an arc of length $s = R\,\theta$; so, providing there is no slippage, the angular displacement of the wheel θ scales to the linear displacement of the mass by the factor R.

Therefore, because their heights are directly proportional, the slope of the θ-t graph, (angular velocity ω) scales to the slope of the s-t graph (linear velocity v) by the same factor R.



Similarly, the slope of the ω-t graph, (angular acceln α) scales to the slope of the v-t graph (linear acceln a) by the factor R.
$\Rightarrow a = R\,\alpha$



(f) Expressions (b), (d) and (e) give $\alpha = \frac{TR}{I}$, $a = \frac{mg - T}{m}$ and $a = R\alpha$.

The answer tells us to eliminate α and T.

(d) is rewritten as $T = mg - ma$.

Put that into (b) and rearrange to get $I\alpha = (mg - ma)R$

Multiply by R to get $IR\alpha = (mg - ma)R^2$.

Use (e) to get $Ia = (mg - ma)R^2$

$\Rightarrow\ Ia = mgR^2 - maR^2$

$\Rightarrow\ Ia + maR^2 = mgR^2$

$\Rightarrow\ a(I + mR^2) = mR^2.g$

$\Rightarrow\ a = \frac{mR^2}{I + mR^2}g$

(g) According to (f), the acceleration of the mass is a fixed fraction of the free-fall acceleration g and is therefore uniform. An object moving a distance d from rest with a uniform acceleration a has a speed v given by the kinematic equation

$$v^2 = 0^2 + 2ax$$
$$= 2\left[\frac{mR^2}{I + mR^2}g\right]x$$
$$\Rightarrow v = \sqrt{\frac{2mR^2gx}{I + mR^2}}$$

NB This can also be done using GPE $\rightarrow$ linear KE + rotational KE and linking $v = R\omega$

Question 3

(a) Since N and n are being considered to have no KE, they are at rest with zero initial momentum; then as momentum is conserved, the total momentum of the proton and carbon nucleus is also zero: $P_C + P_p = 0$

Conservation of mass-energy ⇒ $m_N + m_n = m_C + m_p + Q/c^2$, using Einstein's $E = mc^2$

(b)
$$Q = \Delta mc^2$$
$$= 1.08 \times 10^{-30}\ \text{kg} \times (3 \times 10^8\ \text{ms}^{-1})^2$$
$$= 9.72 \times 10^{-14}\ \text{J}$$
$$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}$$
$$\text{so } Q = 0.6075\ \ \text{MeV}$$

(c) KE and momentum are related by $E_k = \dfrac{p^2}{2m}$.

Since the carbon nucleus and the proton have the same-sized momentum, each has a KE inversely proportional to its mass. With 14 times the mass, the carbon has $^1/_{14}$ the KE of the proton; ie, the proton has $^{14}/_{15}$ and the carbon $^1/_{15}$ of the total KE. The fraction $^1/_{15}$ is about 7%, so for 5% accuracy we cannot assume the carbon remains at rest while the proton has all the KE.

⇒ proton KE = $^{14}/_{15} \times Q$ = $^{14}/_{15} \times 9.72 \times 10^{-14}$ J = 9.07×10^{-14} J.

$$\text{Kinetic energy } E_k = \tfrac{1}{2}mv^2 \Rightarrow \text{speed } v = \sqrt{\frac{2E_k}{m}}$$
$$= \sqrt{\frac{2 \times 9.07 \times 10^{-14}\ \text{J}}{1.67 \times 10^{-27}\ \text{kg}}}$$
$$= 1.04 \times 10^7\ \text{ms}^{-1}$$

(d) From (a), the proton and carbon nucleus have equal-sized momentums

$$\Rightarrow \text{speed of carbon nucleus, } v_C = \frac{m_p}{m_C} v_p$$
$$= \frac{1}{14} \times 1.04 \times 10^7\ \text{ms}^{-1}$$
$$= 7.44 \times 10^5\ \text{ms}^{-1}$$

(e) Initially, 4000 electrons are counted in a given time. After 1 half life 2000 electrons are counted in the same time. After 2 half lives 1000 electrons, and after 3 half lives 500 electrons.

The sample is therefore 3 half lives old = 3 x 5600 years = 16800 years old.

Question 4
Doppler And Blood Flow

(a) The frequency of the reflected wave would be greater than the frequency of the incident wave.

This is because as the blood moves towards the detector the reflected wavefronts will become closer together. The velocity of the ultrasound is unchanged so it will have a higher frequency.

(b) Student 1 has derived the correct equation. $\Delta f = \frac{2fv}{c}\cos\theta$

Student 2's equation $\Delta f = \frac{2fc}{v}\cos\theta$ is clearly incorrect

because with *v* in the denominator, a faster blood flow gives a smaller Doppler shift, which is just wrong.

Student 3's equation $\Delta f = \frac{2fv}{c}\left(1 - \cos\theta\right)$ is incorrect

because we would expect maximum Doppler shift when the blood is moving towards or away from the detector and zero shift when the blood is moving perpendicularly to the sound wave. The $(1 - \cos\theta)$ factor in this equation gives the opposite effect.

(c) Student 1's equation rearranges as:

$$v = \frac{\Delta fc}{2f\cos\theta} = \frac{3100\ \text{Hz} \times 1.5 \times 10^3\ \text{ms}^{-1}}{2 \times 5 \times 10^6\ \text{Hz} \times \cos 30^\circ} = 54\ \text{cms}^{-1}$$

If you use student 2's equation, you will get a very large velocity, which you should realise is unrealistic and therefore try one of the other equations.

If you use student 3's equation, you will get about 350 cms^{-1}. Blood flowing from head to foot in about half a second? Unlikely.

(d) This method measures the magnitude of the **change** in frequency.

There is no indication of whether the change in frequency is positive or negative so therefore there is no indication of whether the blood is flowing towards or away from the detector.