Question 1

a)

The centre of mass will continue in the same direction regardless of the explosion. There is no external force applied to alter the motion of the centre of mass.

b) Assume a spherical asteroid. Take $r = 0.725 \times 10^6$ m (half the "diameter" of Texas).Mass of the asteroid = $4/3\ \pi\ r^3\ \rho$

$= 4.8 \times 10^{21}$ kg

Mass of each half = 2.4×10^{21} kg

Assume all of the bomb energy is converted into KE of the lumps in the *y* direction – velocity (v) in the y direction given by $\frac{1}{2}mv^2 = E$ (assume half the E_k goes to each lump)

$0.5 \times 2.4 \times 10^{21}\ v^2 = 2.5 \times 10^{18}$

$v = 0.046$ m s^{-1}

In 4 hours, 4 × 3600 s, the *y* distance moving by each half will be 657 m (700 m)

The distance needed to be moved is much larger than this so we are doomed!

c) Assume that both halves are spherical

The radius of each spherical lump is given by

$$r^3 = \frac{3M}{4\pi\rho}$$

$$= \frac{3 \times 2.4 \times 10^{21}}{4\pi \times 3000}$$

$$r = 0.58 \times 10^6 \text{ m}$$

Force of gravitational attraction between the two lumps

$$F = \frac{GmM}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 2.4 \times 10^{21} \times 2.4 \times 10^{21}}{1.33 \times 10^{12}}$$

$$F = 2.9 \times 10^{20}$$

This force will cause the two lumps to accelerate towards each other

$$a = \frac{F}{m}$$

$$a = \frac{2.9 \times 10^{20}}{2.4 \times 10^{21}}$$

$$a = 0.12\,\text{ms}^{-2}$$

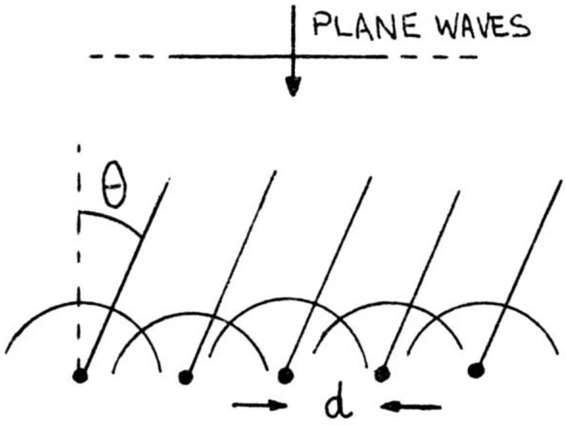
This is 12 cm s^{-2}. The maximum speed of separation from the blast is only

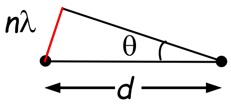
4.6 cm s^{-1}. In other words the self gravitation would almost immediately overwhelm the force from the blast and bring the asteroid back together. The asteroid might "heave" but it would not split apart.

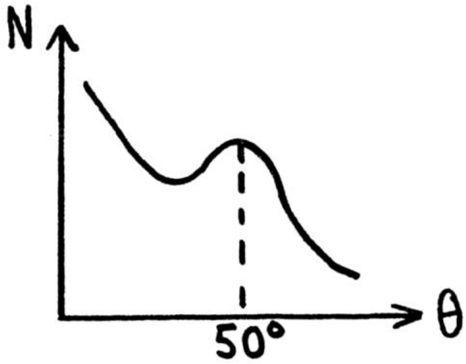
Question 2

(a) Constructive interference in direction θ
$\Rightarrow$ path difference between adjacent sources has to be a whole number of wavelengths $n\lambda$ = side opposite angle θ

$\Rightarrow n\lambda = d\sin\theta$







(b) First order maximum $\Rightarrow n = 1$ at $\theta = 50°$
$\Rightarrow$ wavelength λ is such that $1 \times \lambda = d\sin\theta$
$= 2.15 \times 10^{-10}$ m $\times \sin 50°$
$\Rightarrow$ wavelength $\lambda = 1.65 \times 10^{-10}$ m.

(c) These electrons seem to behave as a wave with wavelength $\lambda = 1.65 \times 10^{-10}$ m.

(d) (i) Assuming near-enough zero initial speed of electrons in electron gun before they are accelerated, then KE gained from rest is loss of electric PE, which is electron charge e $\times$ accelerating voltage V $\Rightarrow E_k = eV$.

(ii) A particle of mass m with speed v has KE $E_k = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$

where $p = mv$ is its momentum,

$\Rightarrow p^2 = 2mE_k = 2meV$ from (i) $\Rightarrow p = \sqrt{2meV}$

(e) electron momentum is $p = \sqrt{2meV} = \sqrt{2 \times 9.11 \times 10^{-31}\ \text{kg} \times 1.6 \times 10^{-19}\ C \times 54\ \text{V}}$

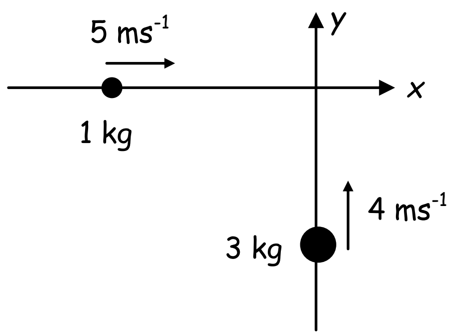
$= 3.97 \times 10^{-24}$ kgms^{-1}

$\Rightarrow$ electron wavelength is $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\ \text{Js}}{3.97 \times 10^{-24}\ \text{kgms}^{-1}} = 1.67 \times 10^{-10}$ m.

This answer compares “pretty nearly” with the answer from (b), so yes, de Broglie’s idea does stand up to the experimental test of Davisson and Germer

Question 3

(a) The x-component of the system's total momentum is just the momentum of the 1 kg object
= mass × x-component of velocity
= 1 kg × 5 ms^{-1} = 5 $kgms^{-1}$.



Similarly, the y-component of the system's total momentum is just the momentum of the 1 kg object = mass × y-component of velocity
= 3 kg × 4 ms^{-1} = 12 $kgms^{-1}$.

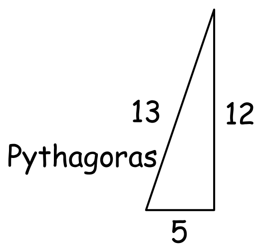
NB: we can use column vectors to write the momenta as $\begin{pmatrix}5\\0\end{pmatrix}$ $kgms^{-1}$ and $\begin{pmatrix}0\\12\end{pmatrix}$ $kgms^{-1}$ so the total momentum is $\begin{pmatrix}5\\12\end{pmatrix}$ $kgms^{-1}$

(b) The total momentum of the system is its total mass × the velocity of its centre of mass
⇒ CM velocity is total momentum / total mass = $\begin{pmatrix}5\\12\end{pmatrix}$ $kgms^{-1}$ / (1 + 3) kg = $\begin{pmatrix}1.25\\3\end{pmatrix}$ ms^{-1}.

The x-component of the CM velocity is 1.25 ms^{-1}, and the y-component is 3 ms^{-1}.

(c) Total KE = KE of 1 kg object + KE of 3 Kg object, $E_k = \frac{1}{2}mv^2$
= $\frac{1}{2}$ × 1 kg × (5 $ms^{-1})^2$ + $\frac{1}{2}$ × 3 kg × (4 $ms^{-1})^2$
= 12.5 J + 24 J
= 36.5 J

(d) After the collision, we have a 4 kg mass moving with momentum $\begin{pmatrix}5\\12\end{pmatrix}$ $kgms^{-1}$,
⇒ a momentum of magnitude 13 $kgms^{-1}$.



KE can be calculated from momentum by

$$E_k = \frac{p^2}{2m} = \frac{(13\ kgms^{-1})^2}{2 \times 4\ kg} = \frac{169}{8}\ J = 21.125\ J$$

⇒ the collision is inelastic, KE is lost to heat in the collision.

(e) Since the motion of the centre of mass is not changed by the collision (total momentum is conserved) and the masses have stuck together, the x and y velocity components of the combined mass are the same as the x and y velocity components of the centre of mass of the system,
⇒ the x-component of the velocity of the combined mass is 1.25 ms^{-1},
and the y-component is 3 ms^{-1} – same as answer (b).

(f) To view the interaction from a frame of reference in which the centre of mass is at rest, we subtract the CM velocity from the given velocities of the masses,

$\Rightarrow$ velocity of 1 kg mass relative to CM is $\begin{pmatrix}5\\0\end{pmatrix}$ ms^{-1} - $\begin{pmatrix}1.25\\3\end{pmatrix}$ ms^{-1} = $\begin{pmatrix}3.75\\-3\end{pmatrix}$ ms^{-1}; and

velocity of 3 kg mass relative to CM is $\begin{pmatrix}0\\4\end{pmatrix}$ ms^{-1} - $\begin{pmatrix}1.25\\3\end{pmatrix}$ ms^{-1} = $\begin{pmatrix}-1.25\\1\end{pmatrix}$ ms^{-1}.

(g) The two masses have speeds relative to the CM before the collision of:

1 kg mass: $\sqrt{3.75^2 + (-3)^2} = \sqrt{23.0625}$ ms^{-1}, and

3 kg mass: $\sqrt{(-1.25)^2 + 1^2} = \sqrt{2.5625}$ ms^{-1}.

Total KE relative to the CM before the collision is therefore

$\frac{1}{2} \times 1$ kg $\times$ ($\sqrt{23.0625}$ ms^{-1})^2 + $\frac{1}{2} \times 3$ kg $\times$ ($\sqrt{2.5625}$ ms^{-1})^2

= 11.53125 J + 3.84375 J

= 15.375 J

NB The total KE relative to the CM after the collision is zero so the KE lost to heat in the collision is 15.375 J, which happens to be the difference between answers (c) and (d).

Observers in different frames see different KEs for the masses before and after the collision but they see the same loss in total KE.

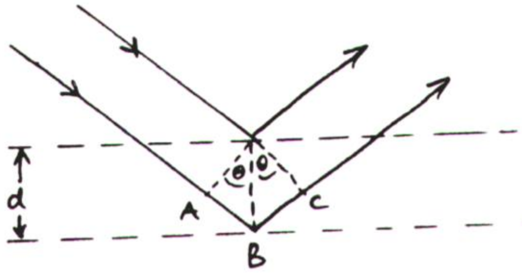
Question 4

Bragg's Law

(a) For interference fringes to be produced by two sources of light at a distance d apart, the sources must have:

1 the same wavelength
2 fixed phase difference or coherent
3 a separation, *d*, greater than the wavelength
4 comparable amplitudes

(b) The extra path difference for the lower ray relative to the upper ray is AB + BC =2AB but AB/*d* = sinθ



Therefore for a maximum in the reflected intensity at angle θ, the path difference must be an integral number of wavelengths:

$m\lambda = 2d\sin\theta \qquad m = 1, 2, 3, ...$

(c) 3rd order $\Rightarrow m = 3$

$$\theta = \frac{29.2°}{2} = 14.6°$$

$$\lambda = 1.27 \times 10^{-10}\ m$$

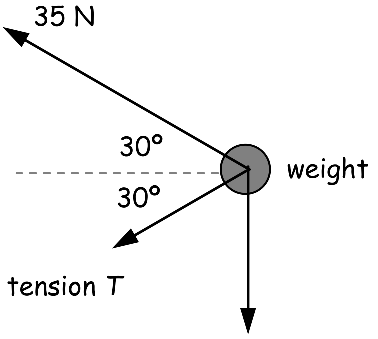
$$\Rightarrow d = \frac{m\lambda}{2\sin\theta} = \frac{3 \times 1.27 \times 10^{-10}}{2 \times \sin 14.6°}$$

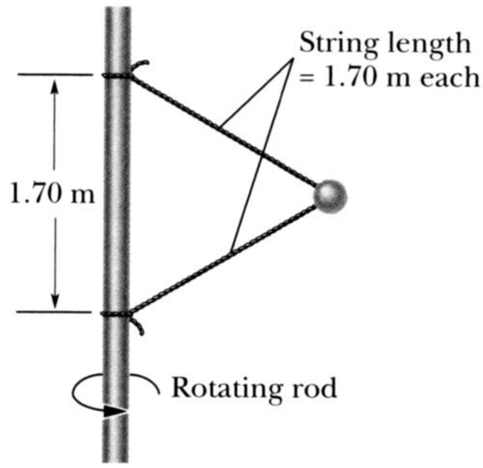
$$= 7.56 \times 10^{-10}\ m$$

(d) Visible light has a wavelength of about 500 nm, or 5000×10^{-10} m, which is too large for interference to be observed from adjacent planes.

Question 5

(a) A free-body diagram shows the forces exerted on a object by other bits of the universe elsewhere. There are three forces acting on the ball – a weight and two tensions.





(b) The rod and strings make an equilateral triangle so the strings are angled at 30° to the horizontal.
The ball is constrained to move in a horizontal circle; it has therefore zero vertical acceleration and so the total of all the **vertical** components of the forces acting on the ball is zero.

The vertical component of the weight is 1.34 kg × 9.8 Nkg^{-1} = 13.1 N down.

The vertical component of the upper tension is 35 N × sin 30 ° = 17.5 N up.

The vertical component of the lower tension is $T \times$ sin 30 ° = 0.5T N down

® 0.5T + 13.1 = 17.5tension

® tension $T = (17.5 - 13.1)/0.5 = 8.8$ N

(c) The total force on the ball is the sum of the horizontal components of the forces acting (+ the sum of the vertical components, which is zero)

® F = 35 N × cos 30 ° + 8.8 N × cos 30 ° = 37.9 N

(d) The centripetal acceleration of the ball in its rotation is caused by the total force F acting on it, ie, 37.9 N horizontally towards the rod.

$$\Rightarrow F = \text{mass} \times \text{centripetal accel}^{n} = \frac{mv^2}{r}$$

$$37.9\text{ N} = \frac{1.34\text{ kg} \times v^2}{1.70\text{ m} \times \cos 30^0} \Rightarrow \text{ speed } v = \sqrt{\frac{37.9 \times 1.70 \times \cos 30^0}{1.34}} = 7.5\text{ ms}^{-1}$$

Question 6

a) Each bulb will have 12 V across it due to symmetry – given that this is the operating voltage of each bulb the resistances can be calculated by using $R=V^2/P$ (the 20W bulbs have resistance = 7.2 ohms and the 40 W bulbs have resistance = 3.6 ohms). The majority of the current will take the path of least resistance so current will go from b to a.

$$I_1(\text{top left hand branch}) = \frac{12}{7.2}$$

and

$$I_2(\text{bottom left hand branch}) = \frac{12}{3.6}$$

$$I_{ba} = -I_1 + I_2 = 1.67\text{A}$$

b)

When switch S_2 is opened the same current exists in L_1 and L_2. The resistance of L_2 is unknown as we do not know the V-I characteristics of the bulb but we can assume the value of the $L_2 > L_1$. This means L_1 has more voltage across it and therefore an increased current and brightness (it may blow). This then means that L_2 has less voltage than before and therefore less current so will be dimmer.

c)We would need the V-I characteristics. Essentially its resistance values for all V and I values.